Single Pure – Integration By Parts

Remember: 'Integrate the thing you are going to integrate, leave the other thing alone, take away the integral of "do both"'.

1. Evaluate

   (a) $\int xe^x\,dx$. $\boxed{(x-1)e^x+c}$

   (b) $\int 2xe^{3x}\,dx$. $\boxed{\frac{2xe^{3x}}{3}-\frac{2e^{3x}}{9}+c}$

   (c) $\int x\sin 2x\,dx$. $\boxed{\frac{\sin 2x-2x\cos 2x}{4}+c}$

   (d) $\int (1-kx)e^x\,dx$. $\boxed{e^x(1+k-kx)+c}$

   (e) $\int \ln x\,dx$. $\boxed{x(\ln x-1)+c}$

   (f) $\int \theta\cos(2\theta-\pi)\,d\theta$. $\boxed{\frac{2\theta\sin(2\theta-\pi)+\cos(2\theta-\pi)}{4}+c}$

   (g) $\int \theta\cos n\theta\,d\theta$. $\boxed{\frac{\theta\sin n\theta}{n}+\frac{\cos n\theta}{n^2}+c}$

   (h) $\int 3x^2e^{2x}\,dx$. $\boxed{\frac{3e^{2x}}{4}(2x^2-2x+1)+c}$

   (i) $\int ax^n\ln x\,dx$. $\boxed{\frac{ax^{n+1}(n\ln x+\ln x-1)}{(n+1)^2}+c}$

   (j) $\int ay^2\cos by\,dy$. $\boxed{\frac{ay^2\sin by}{b}+\frac{2ay\cos by}{b^2}-\frac{2a\sin by}{b^3}+c}$

   (k) $\int (\ln x)^2\,dx$. $\boxed{x((\ln x)^2-\ln x+1)+c}$

   (l) $\int \sqrt[3]{x}\ln(4x)\,dx$.

2. A litte harder here…

   (a) $I=\int e^x\sin x\,dx$. $\boxed{\frac{e^x(\sin x-\cos x)}{2}+c}$

   (b) $I=\int e^y\cos y\,dy$. $\boxed{\frac{e^y(\cos y+\sin y)}{2}+c}$

   (c) $I=\int e^{kx}\sin x\,dx$. □

   (d) $I=\int e^{kx}\cos px\,dx$. $\boxed{\frac{e^{kx}(p\sin px+k\cos px)}{k^2+p^2}+c}$

3. I *definitely* like these…

   (a) $\int_0^1 \frac{2x}{e^x}\,dx$. □

   (b) $\int_0^{\pi} 2\theta\sin\theta\,d\theta$. $\boxed{2\pi}$

   (c) $\int_2^3 -xe^{x+1}\,dx$. □

   (d) $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \theta\cos 2\theta\,d\theta$. $\boxed{\frac{3\pi-\sqrt{3}\pi-3}{24}}$

   (e) $\int_1^2 2x\sqrt{x-1}\,dx$. [Even though a substitution is better here, please use parts.] □

   (f) $\int_{\pi}^{2\pi} 3\theta^2\sin\left(\frac{\theta}{2}\right)d\theta$. $\boxed{24(\pi^2-\pi-2)}$

4. It is given that $\frac{dy}{dx}=x^3e^{2x}$. Find the equation of the curve if it passes through the point $(0,3)$. □

5. Find the area bounded by the curve $y=axe^{bx}$, the line $x=1$ and $x=2$. □

6. Find the volume formed when the curve $y=x\sqrt{\sin x}$ is rotated about the x-axis between $x=0$ and $x=\frac{\pi}{2}$. □